

One-Dimensional Stability of AP Deflagrations

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A small-perturbation investigation of ammonium perchlorate deflagrations is made to determine the intrinsic stability of a one-dimensional burning configuration. It is found on the basis of a particular model of the flame structure that above a certain pressure, dependent upon the numerical values used, a stable one-dimensional flame cannot exist. Detailed calculations are presented for a preferred set of parameters, and the pressure above which stability is impossible is between 620 and 2140 psia, depending on whether the AP-gas interface is in equilibrium or is pyrolysis controlled.

Nomenclature

A	= defined in Eqs. (24)
B'	= defined in Eqs. (25)
B	= chemical kinetics frequency factor
b	= pyrolysis rate constant of Eq. (13)
c_s, c_p	= specific heat of the AP and gas at constant pressure, respectively
E_s	= pyrolysis activation energy
g, g_a	= dimensionless temperature in the solid phase and gas phase, respectively
g_s, g_{sf}	= dimensionless surface temperature and flame temperature
G	= dimensionless temperature perturbation function
k	= reaction time constant
Le	= Lewis number
L	= endothermic heat of dissociative sublimation
p	= pressure
p_{F_s}	= partial pressure of NH_3 at the interface
q_s, q_R	= endothermic heat of dissociative sublimation and exothermic heat of gas phase reaction divided by $c_p T_0$, respectively
R	= perturbation in burning rate or universal gas constant
\mathcal{R}	= r/\bar{r}
r	= burning rate
s	= $s_r + is_i$ = complex eigenvalue for intrinsic oscillations
t	= time
T	= temperature
x	= coordinate normal to interface
u	= axial velocity
Y	= mass fraction
y	= interface mass fraction perturbation
Z	= perturbation in ζ
α	= thermal diffusivity
β	= defined by Eqs. (23)
ϵ_i	= E_s/RT_0 or L/RT_0 if $i = s$ or L , respectively
λ	= thermal conductivity or $\frac{1}{2}[1 + (1 + 4s)^{1/2}]$
η	= c_s/c_p
ξ	= $\lambda_s c_p / \lambda_g c_s$
τ_c	= dimensionless reaction time
τ_c^*	= $1/\rho_0 B$
ρ	= density
ζ	= $2Y_{F_s}^2 \rho_0 / (\tau_c \rho_s \mathcal{R}^2 \xi)$

Subscripts

f	= flame
F	= NH_3
g	= gas phase
0	= cold AP
s	= interface or solid phase

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Introduction

MANY theories have been advanced in recent years concerning the response of solid-propellant burning rates to pressure oscillations imposed in the chamber gases. An excellent review of these theories and their comparison with experiment is given by Culick.¹ Concurrently, as an aid to understanding combustion behavior in AP composite propellants, experimental work has been carried out in pure AP deflagrations.² A composite of several workers' data on the mean deflagration rate as a function of pressure is given in Fig. 1. It is interesting, therefore, that the stability of the AP deflagration in a one-dimensional sense has not been investigated, since it would also apparently have bearing on the behavior of composite propellants. This stability is examined in the present paper.

There are two reasons for the stability investigation, in addition to the one given concerning its relevance to solid propellant combustion. First, there has never been a satisfactory theoretical explanation given for the observed deflagration limit at roughly 300 psi. Secondly, it is known that the deflagration ceases to be macroscopically steady and one-dimensional above about 2000 psi.² The work in this paper was originally intended to shed some light on either or both of these phenomena; however, the results are directly

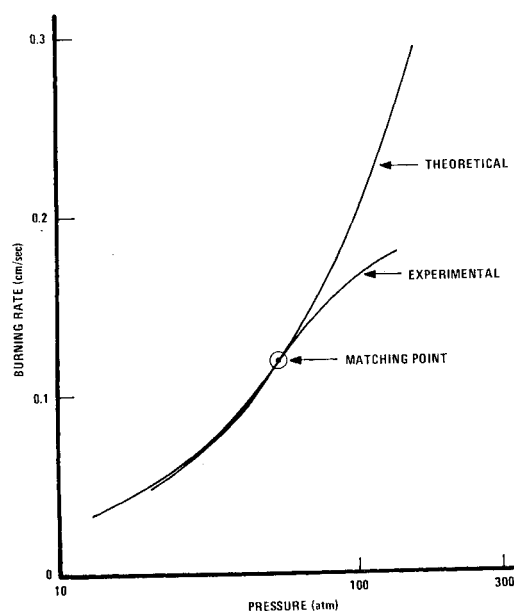


Fig. 1 Theoretical and experimental deflagration rate as function of pressure.

Table 1 Values used in calculations

T_0	= 300°K
c_p	= 0.3 cal/g°K
c_s	= 0.275 cal/g°K
\bar{r}	= 0.3 in./sec at $p = 800$ psi with $T_s = 800^\circ\text{K}$
$q_s c_p T_0$	= 513 cal/g
$q_{RC} T_0$	= 798 cal/g
E_s	= 30 kcal/mole AP
L	= 29 kcal/mole NH_3
λ_g	= 2×10^{-4} cal/sec°K cm
λ_s	= 1.19×10^{-3} cal/sec°K cm
ρ_s	= 1.95 g/cm ³
ρ_g	= determined by perfect gas law at T_s and with a molecular weight of 27.95 g/mole

relevant only to the second. It will be seen, however, that there is a possibility of relevance concerning the lower deflagration limit. It is found theoretically that above a certain pressure a stable one-dimensional deflagration cannot exist.

In order to investigate the configuration stability, a model of the burning process must be constructed or chosen from the literature. Usual treatments, summarized by Culick,¹ are inadequate for AP, and, consequently, a new model has been developed. The model contains controversial elements, but it at least presents a self-consistent approach.

The approach taken is to investigate the possibility of intrinsic oscillations, either damped or amplified, of the flame. This is equivalent to an investigation of the poles of the burning-rate response function when the propellant is acted upon by an external gas phase oscillation. An unstable pole is one that allows an intrinsic oscillation to grow without limit in amplitude. Such unstable behavior has been recently found theoretically for composite propellants.³ This approach has also often been taken in the past.¹

Model of the AP Flame

Although there are several uncertainties regarding the precise structure of the AP decomposition flame, there appears to be enough information to construct a self-consistent theory that explains many of the observable features, at least in the pressure regime of nearly planar steady deflagration. Several mechanistic and data choices will now be made; literature references are cited to support these choices, but controversy surrounds some of them.

In the solid phase constant thermal properties are chosen⁴ as listed in Table 1. These properties are assumed the same if the AP is either in the orthorhombic or cubic phase and regardless of the temperature level. The heat of phase transition is taken as an endothermic 19.4 cal/g, based on measurements made by differential scanning calorimetry at this laboratory. The endothermic dissociative sublimation step, which is assumed to occur $[\text{NH}_4\text{ClO}_4(\text{s}) \rightarrow \text{NH}_3(\text{g}) + \text{HClO}_4(\text{g})]$ at the solid gas interface, is assigned a value of 58 kcal/g-mole or 493.6 cal/g.⁵ Treatment of the heat transfer in the solid phase assumes that the heat of transition occurs at the solid gas interface in order to avoid a treatment of an interior heat source. The effective over-all sublimation heat is therefore taken as an endothermic 513 cal/g.

The equation for heat transfer in the solid phase may be written as

$$\rho_s c_s \partial T / \partial t^* + \rho_s c_s \bar{r} \partial T / \partial x^* = \lambda_s \partial^2 T / \partial x^{*2} = \alpha_s \rho_s c_s \partial^2 T / \partial x^{*2} \quad (1)^\dagger$$

where the origin of coordinates is always fixed at the solid-gas interface. Equation (1) is made dimensionless by introducing a characteristic temperature T_0 , the cold solid temperature, a characteristic time α_s / \bar{r}^2 , and a characteristic length α_s / \bar{r} .

[†] Superscript bar denotes the steady state. Superscript* denotes a dimensional quantity.

There is obtained

$$\partial^2 g / \partial x^2 = \alpha \partial g / \partial x + \partial g / \partial t \quad (2)$$

which holds in the steady and unsteady states.

The interface condition on heat transfer states that heat transfer from the gas phase goes toward supply of the sublimation heat and heat transfer into the solid. After nondimensionalization and assuming $\rho_s \gg \rho_g(0, t)$, the interface conditions on mass flow and heat transfer are

$$\begin{aligned} \frac{1}{\xi} \frac{\partial g_g}{\partial x} \bigg|_{0,t} &= \alpha \bar{q}_s + \eta \frac{\partial g_s}{\partial x} \bigg|_{0,t} \\ \rho_s \bar{r} &= (\rho_g u)_{0,t} \end{aligned} \quad (3)$$

where the sublimation heat is made dimensionless by $c_p T_0$.

In Eq. (3) parameters have entered for the gas phase. They are c_p and λ_g . λ_g is chosen as 2×10^{-4} cal/sec°K cm corresponding roughly to that of N_2 at 1200°K; c_p is chosen as 0.3 cal/g°K. Observed reaction heats are substantially lower than theoretical values, and the mechanism of heat loss or inefficiency is unknown; consequently it cannot be properly accounted for in the theoretical model. It appears most rational to use a heat of reaction based on observed products. The value for the exothermic heat of gas phase reaction is picked as 798 cal/g. This yields an over-all heat of reaction as an exothermic 275 cal/g which is the observed value at 72 atm; using the chosen value of c_p this yields a flame temperature of 1218°K in accordance with observations.^{5,6} It should be cautioned, however, that the observed reaction heat is pressure dependent,⁶ and this will cause some uncertainty in the results, particularly with respect to the relevance of this work at the low-pressure deflagration limit.

At this point the quasi-steady approximation is introduced for the gas phase. Time derivatives are neglected in comparison with spatial derivatives and field quantities. This approximation is common in solid-propellant stability theory and is even more valid here because of the small flame standoff distance. In Eqs. (3), therefore, continuity demands $\rho_g u_{0,t} = \rho_g u_{x,t}$. It is also fashionable to assume spatially constant values of λ_g and ρ_g . This will also be assumed here, but it is a much more valid assumption for AP deflagration because of the small temperature difference between the interface and flame. Spatially constant λ_g and ρ_g assumptions may properly be regarded as a first approximation in a valid perturbation procedure. The Shvab-Zeldovich formulation for the gas phase problem is adopted, since it is a reasonable approximation for the case at hand.⁷ Differences in diffusion coefficient between NH_3 and HClO_4 are ignored. The one-step over-all reaction assumed is $\text{NH}_3(\text{g}) + \text{HClO}_4 \rightarrow i$ Products. The energy equation becomes after nondimensionalization

$$\partial^2 g_g / \partial x^2 - \alpha \xi \partial g_g / \partial x = -2q_R \xi (\rho_g / \rho_s) Y_F^2 / \tau_c \quad (4)$$

The reaction rate term in Eq. (4) contains the factor 2 because \bar{q}_R is based on a unit mass of NH_4ClO_4 , but the reaction rate is written on the basis of the production rate of NH_3 . Here NH_3 and HClO_4 are treated as indistinguishable molecules with the same molecular weight; and τ_c is the chemical time which would ordinarily contain a pre-exponential factor and an activation energy. The activation energy will be chosen as zero in accordance with previous arguments.^{8,9} Further remarks concerning this assumption will follow. Thus, $\tau_c = \bar{r}^2 / (\alpha_s \rho_g B)$ is a dimensionless kinetic time.

The equation for transport of NH_3 thus becomes

$$\begin{aligned} \partial^2 Y_F / \partial x^2 - (\alpha \xi / Le) \partial Y_F / \partial x &= \\ &= -(1/Le) \xi (\rho_g / \rho_s) Y_F^2 / \tau_c \quad (5) \end{aligned}$$

and a constant Lewis number will be assumed. The general solution to Eqs. (4) and (5) cannot be written because of the nonlinear reaction rate term. A great many approximate techniques have been developed in the past, usually relying

upon the fact that activation energies are large, so that reaction is important only in the last part (large x) of the transport zone. This procedure has also been used for the AP flame.⁸ Under the assumption of low activation energy reaction must be spread throughout the diffusion zone. Therefore, the method chosen for solution of Eqs. (4) and (5) is by iteration; a form for the right-hand sides is selected containing an arbitrary constant, and the form is chosen to approximately describe the actual reaction path. It is assumed that

$$Y_F = Y_{F_s} \exp(-ax) \quad (6)$$

may be inserted in the right-hand sides of Eqs. (4) and (5). The arbitrary constant a is selected, such that the integrated disappearance rate of the fuel is equal to the flow rate from the surface; i.e.,

$$\frac{1}{2} \rho_o u = \rho_o^2 B \int_0^\infty Y_F^2 dx$$

which yields $a = \rho_o B Y_{F_s}^2 / u$. Equations (4) and (5) may now be integrated. A further iteration could be attempted, but this first trial is all that is considered necessary at this point because of other uncertainties in the computations. Note that Eq. (6) yields the highest reaction rate at the surface where the $\text{NH}_3 \cdot \text{HClO}_4$ collision rate is the highest. The boundary conditions on Eqs. (4) and (5) are that the mass transfer rate of fuel vapor at the interface must equal $\frac{1}{2}$ the total burning rate, and that at infinity the fuel must be consumed.

$$\left. \frac{\partial Y_F}{\partial x} \right|_{x=0} = \frac{\alpha \xi}{Le} \left(Y_{F_s} - \frac{1}{2} \right) \quad (7)$$

$$Y_F|_{x=\infty} = 0$$

Furthermore, the temperature must reach the adiabatic flame temperature

$$g_o(\infty, t) = g_{o_f} = g_s + \bar{q}_R - \frac{1}{\alpha \xi} \left. \frac{\partial g_g}{\partial x} \right|_{0,t} \quad (8)$$

$$g_o(0, t) = g_s$$

Integrating Eq. (5) subject to Eqs. (7) yields an equation for Y_{F_s} .

$$2Y_{F_s}^3 [Le / \tau_c (\rho_o / \rho_g) \alpha^2 \xi] + Y_{F_s} = \frac{1}{2} \quad (9)$$

Integrating Eq. (4) subject to Eqs. (8) yields the heat transfer from the gas to the solid phase

$$\left. \frac{\partial g_g}{\partial x} \right|_{0,t} = \alpha \xi \bar{q}_R \left[\frac{\zeta}{\zeta + 1} \right] \quad (10)$$

with $\zeta = 2Y_{F_s}^2 \rho_o / (\tau_c \rho_s \alpha^2 \xi)$. With this variable Eq. (9) may be written

$$Y_{F_s} (1 + \zeta Le) = \frac{1}{2} \quad (11)$$

To complete the solution Eq. (2) must be integrated subject to

$$g(0, t) = g_s \quad (12)$$

$$g(-\infty, t) = 1$$

and g_s is still to be determined. Actually, Eqs. (12) are not sufficient. In the fully unsteady problem initial conditions or some other condition relative to temporal behavior (such as periodicity) must be added. The treatment here will be concerned with small perturbations about a steady-state condition, and these small perturbations, resulting in a linear problem, will be assumed to have the time dependence e^{st} where s is a complex number $s_r + is_i$. At any rate the solution to Eqs. (2) will yield $\partial g / \partial x|_{0,t}$ as a function of $\alpha(t)$ and $g_s(t)$. Then Eq. (3) becomes a relation between α , ζ , and g_s , presuming ξ and η are known. Since a relation exists by definition between ζ , Y_{F_s} and α , presuming the behavior of τ_c and ρ_o are known, Eq. (11) provides a third equation in four

unknowns ζ , α , Y_{F_s} , and g_s . The missing information comes from the interface condition. Two limiting conditions will be assumed. The first is an unopposed rate process (pyrolysis)

$$r = b \exp(-E_s / RT_s) = b \exp(-\epsilon_s / g_s) \quad (13)$$

In order to determine the steady-state burning rate, which enters the characteristic time used for nondimensionalization, b must be determined. It is calculated under the assumptions that $E_s = 30$ kcal/mole,¹⁰ $r = 0.3$ in./sec at 800 psi² where the surface temperature is 800°K. The surface temperature at this condition is taken as the average figure accepted by a large group of workers in the field of AP deflagration.¹¹ This point is chosen for the match since the slopes of the experimental and theoretical burning rates match best here. Then

$$\alpha = \exp[-\epsilon_s (g_s^{-1} - \bar{g}_s^{-1})] \quad (14)$$

There is current controversy over whether the interface is in a near equilibrium state under high-pressure deflagration conditions¹²; therefore, the opposite limiting extreme is chosen, that of an equilibrium interface. In this case the van't Hoff expression¹³

$$p_{F_s} = c \exp(-L / RT_s)$$

may be used. Here L is chosen as 29 kcal/mole.[†] The equilibrium relation in dimensionless form becomes

$$Y_{F_s} = Y \exp[-\epsilon_L (g_s^{-1} - \bar{g}_s^{-1})] \quad (15)$$

Steady State

Denote steady-state solutions by a bar superscript. Equations (2) subject to Eqs. (12) yield

$$\partial \bar{g} / \partial x|_0 = (\bar{g}_s - 1)$$

Then $\alpha = 1$ and Eqs. (3) with Eq. (10) yield

$$\bar{\zeta} = [\bar{q}_s + \eta(\bar{g}_s - 1)] / [\bar{q}_R - \bar{q}_s - \eta(\bar{g}_s - 1)] \quad (16)$$

At the matching point therefore $\bar{\zeta} = 4.731$, using stated numbers. Assuming a Lewis number of unity, for which there appears no better arbitrary choice, and Eq. (11), $Y_{F_s} = 0.0873$. Then the definition of $\bar{\zeta}$, the known burning rate and physical constants, and the perfect gas law yield the reaction time in terms of pressure, assuming no temperature dependence

$$\tau_c^* = kp^{-1} = 1 / \rho_o B = 5.09 \times 10^{-5} p^{-1} \text{ sec} (p \text{ in atm}) \quad (17)$$

It should be noted that this derived time is one order of magnitude larger than that derived in Ref. 10. This discrepancy is due to the distributed reaction model. At the matching point $\tau_c^* = 10^{-6}$ sec, the flame thickness is roughly $u\tau_c^* \approx 10^{-5}$ cm = 0.1 μ , the mean free path is roughly 3×10^{-7} cm, and the mean free path between NH_3 and HClO_4 molecules is about 3×10^{-6} cm. Furthermore, the collision frequency is near 3×10^{11} sec⁻¹ and the collision frequency for NH_3 molecules is roughly 3×10^{10} sec⁻¹. These magnitudes are consistent with previous arguments concerning the fast kinetics with a relatively low activation energy.⁸⁻¹⁰ Although there are difficulties concerning the acceptance of the present model,¹⁴ at least a self-consistent structure is presented.

The burning rate at any T_s may be computed from Eq. (13). Equation (16) yields $\bar{\zeta}$, Eq. (11) yields Y_{F_s} , and the ζ definition with Eq. (17) yields the pressure corresponding to T_s . The burning rate vs pressure is shown in Fig. 1; both experi-

[†] This is one half the endothermic heat of dissociative sublimation per mole AP. The one-half factor converts this to a per mole basis for NH_3 . The author would like to thank an anonymous reviewer for pointing out an error in this regard in the original manuscript.

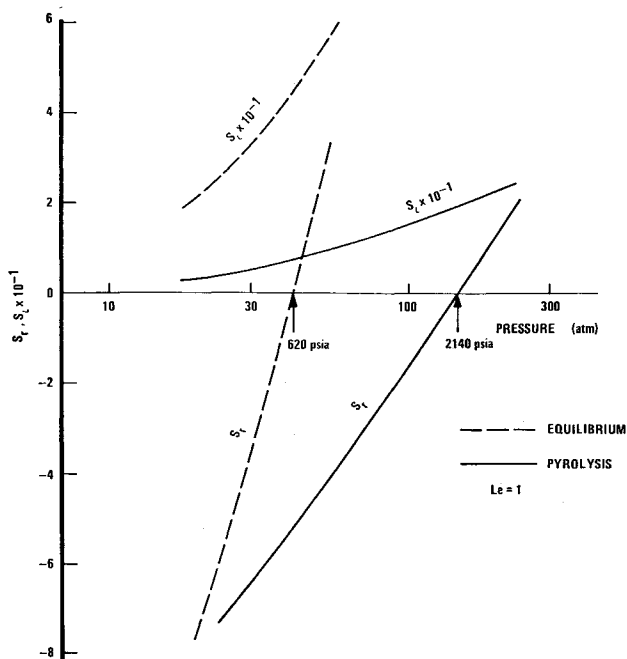


Fig. 2 Eigenvalue for intrinsic oscillations as a function of pressure.

ment and this theory are presented. The burning rate is virtually proportional to pressure because of the weak theoretical dependence on T_s for the large value of ϵ_s .

For an equilibrium interface the matching point computation follows in a similar manner. Since, in Fig. 1, the theoretical curves for nonequilibrium or equilibrium are indistinguishable, only one theoretical curve is presented. This occurs because ϵ_L is large ($\epsilon_L \approx \epsilon_s$).

It should be pointed out that the steady-state model requires only one adjustable constant, b or c ; the rest are supportable by data.

Unsteady State

The intrinsic stability of the assumed steady burning configuration is now investigated. By "intrinsic" is meant that there is no forcing action. That is, no external pressure or temperature oscillations are imposed, as is usually the case in stability calculations for solid propellants. Seeking conditions under which the one-dimensional configuration is unsteady with one-dimensional oscillations, let

$$\alpha = 1 + Re^{st}, Y_{F_s} = \bar{Y}_{F_s} + ye^{st}, g = \bar{g} + Ge^{st} \quad (18)$$

$$\zeta = \bar{\zeta} + Ze^{st}$$

Substitution into Eqs. (2) and (12) yields an equation which is integrable and which has the well known solution

$$\partial G / \partial x)_0 = \lambda G_s + R(\bar{g}_s - 1) / \lambda \quad (19)$$

$$\lambda = \frac{1}{2} [1 + (1 + 4s)^{1/2}]$$

Assuming there are no pressure oscillations allowed and that $\tau_c^* \xi / \rho_0$ is independent of temperature,[§] it follows from definition that

$$Z = 2\bar{\zeta}(y/Y_{F_s}) - 2R\bar{\zeta} \quad (20)$$

From Eq. (11)

$$y(1 + \bar{\zeta}Le) + LeZ\bar{Y}_{F_s} = 0 \quad (21)$$

§ The temperature independence is incorrect, but it introduces negligible differences in the computations.

and from Eq. (10)

$$\frac{1}{\bar{\zeta}} \frac{\partial G_s}{\partial x} \Big|_0 = R\bar{q}_R \left[\frac{\bar{\zeta}}{\bar{\zeta} + 1} + \frac{Z/R}{(\bar{\zeta} + 1)^2} \right] \quad (22)$$

Furthermore, from Eq. (14) for a nonequilibrium interface

$$R = \epsilon_s G_s / \bar{q}_s^2$$

and from Eq. (15) for an equilibrium interface

$$y = \bar{Y}_{F_s} \epsilon_L G_s / \bar{q}_s^2$$

which when combined with Eqs. (20) and (21) yields

$$R = \epsilon_L G_s (1 + 3\bar{\zeta}Le / 2\bar{\zeta}Le) / \bar{q}_s^2$$

Thus, in general,

$$R = \epsilon_s G_s / (\bar{q}_s^2 \beta)$$

$$\text{equilibrium } i = L\beta = 2\bar{\zeta}Le / (1 + 3\bar{\zeta}Le) \quad (23)$$

$$\text{pyrolysis } i = s\beta = 1$$

Combination of Eqs. (3) and (19-23) yields a single equation for R

$$R \left(\frac{A}{\lambda} + \lambda - \frac{\epsilon_s}{\beta \bar{q}_s^2 \eta} \left\{ \bar{q}_R \left(\frac{\bar{\zeta}}{\bar{\zeta} + 1} \right) \left[1 - \frac{2(1 + Le\bar{\zeta})}{(1 + \bar{\zeta})(1 + 3Le\bar{\zeta})} \right] - \bar{q}_s \right\} \right) = 0$$

$$A = [(\bar{q}_s - 1) / \beta \bar{q}_s^2] \epsilon_i \quad (24)$$

In order that R be nonzero the bracketed quantity must be zero. This quantity

$$(A/\lambda) + \lambda - B' = C \quad (25)$$

which defines B' and C is of the same general form as the denominator of many solid propellant admittance functions.¹⁴ If the oscillation were forced, $C = 0$ would correspond to an infinite response. Here, an intrinsic finite oscillation is allowed for $C = 0$ since there is no external forcing. For $C = 0$

$$s_r = (B'/2)(B' - 1) - A \quad (26)$$

$$s_i = [(B'/2) - 1](B' - 1)B'/2$$

and if B' is sufficiently positive relative to A , $s_r > 0$ results, which is an unstable oscillation. This is to be interpreted that if any noise in the combustion system exists at frequencies for which $s_r > 0$, the oscillations at these frequencies will be amplified in time.

Results

Some basic case calculations are presented in Fig. 2. It is seen that depending upon the interface behavior there is a maximum pressure, between 620 and 2140 psia, above which a stable deflagration cannot exist. The frequencies which would be selected at these two points of neutral stability are 27 and 103 Hz, using the theoretical value of burning rate. As pressure and surface temperature rise A falls very slightly, but B' rises rapidly because $\bar{\zeta}$ falls. B' is a sensitive calculation because it is a small difference between two comparable numbers. This is related to the fact, which is unusual in solid deflagrations, that \bar{q}_R and \bar{q}_s are of comparable magnitude. In one particular model of solid propellants the AP portion of the deflagration model led to an unstable pole³; it was found if the "exothermic" surface heat of decomposition were large enough that the unstable pole resulted. That model collapses the AP flame to a much thinner region than the over-all propellant flame and assigns no structure to the

¶ B' is used rather than B in order that there be no confusion with the B function of Ref. 1.

Table 2 Calculation sensitivities for a pyrolysis controlled interface

$\partial \ln p / \partial \ln T_0 = -5.0$	$\partial \ln p / \partial \ln(q_R - q_s) = 7.4$
$\partial \ln p / \partial \ln q_s = 0.3$	$\partial \ln p / \partial \ln c_s = -13$
$\partial \ln p / \partial \ln T_s = -16$	$\partial \ln p / \partial \ln E_s = -3.3$

AP flame. The model here is not the same, however. Although it is true that in the case of the monopropellant flame the heat feedback increases with an increase in burning rate, the entire heat release is not pumped back into the surface. An increase in \bar{q}_R actually is stabilizing.

Notice that an equilibrium interface is less stable than a pyrolysis interface. Since equilibrium is favored as pressure increases there is unconditionally a pressure above which stability cannot exist according to this model.

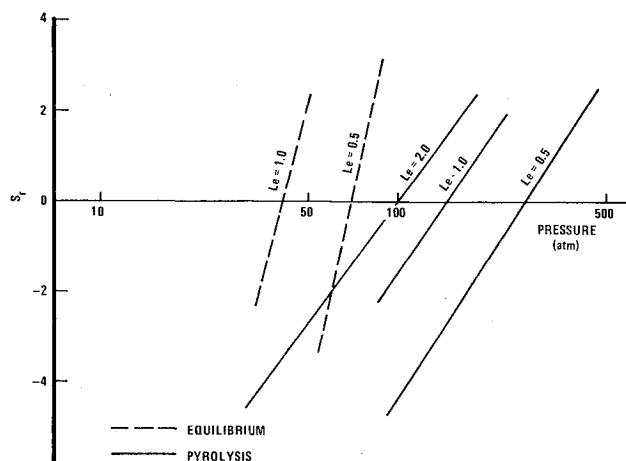
An indication of the calculation sensitivity is shown in Fig. 3 where the effect of Lewis number may be seen. Although the dependence is extreme this is actually one of the weaker sensitivities. Table 2 indicates some further effects of the chosen parameters upon the pressure level for neutral stability ($s_r = 0$). The sensitivities with respect to the AP soak temperature T_0 , the over-all heat of reaction $q_R - q_s$, the solid phase specific heat c_s , and the assumed surface temperature at the matching point T_s are particularly noteworthy. Singled out for special consideration, however, is the dependence upon $q_R - q_s$. If this value is chosen as 245 cal/g instead of 275 cal/g the stability pressure drops to 28 atm for the case of the pyrolysis controlled interface. A substantially lower value would exist for the stability pressure of the equilibrium interface. However, 245 cal/g is the observed value for a deflagration at 1 atm⁶ (burning enforced by raising T_0); that is, the over-all heat of reaction drops as pressure falls. It is entirely conceivable, therefore, that two pressures exist for neutral oscillations; that is, a low pressure deflagration limit might be predicted through stability analysis. Detailed calculations of this phenomenon have to await more information on the actual interface behavior and the detailed behavior of the heat of reaction as functions of pressure.

There is no direct effect on the stability calculation of variations in c_p and λ_p . These parameters do, however, affect the derived value of τ^* . That is, because the procedure for the steady-state calculation is one of matching experimental data at one point, this independence from c_p and λ_p arises. Furthermore, the stability calculation is independent of λ_s ; however, the computation of the dimensional value of frequency from the dimensionless s_r does depend upon λ_s .

It should be remarked that one-dimensional analysis gives a necessary but not sufficient condition for stability. That is, if three-dimensional disturbances were allowed the critical pressure for stability might be lowered, and, in fact, more than one stability limit might be discovered. The present theory does not imply, of course, that a flame cannot exist in the region of instability. This is a linear, one-dimensional theory. Certainly an unsteady, nonlinear, three-dimensional structure could be assumed after breakdown of the steady one-dimensional flame.

Conclusions

1) On the basis of current knowledge of the AP decomposition flame, a one-dimensional model of AP deflagration in the steady and unsteady states has been constructed. It is found that this flame is unstable to one-dimensional disturbances above some pressure of the order of 1000 psi regardless of the details of the interface behavior.

**Fig. 3 Effect of Lewis number on the growth rate.**

2) This calculation is highly sensitive to numerical values assigned to the physical and chemical parameters; however, the qualitative results are unaltered.

3) This instability has been observed, but it is not known if the theoretical and observed mechanisms are the same.

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